SOUND ANALYSIS OF LONGITUDINAL VIBRATIONS OF *QIN* STRINGS

"Silk and *wutong* combine as the *qin*; antique sound emerges from within." [1] Sound quality of the *qin* depends not only on the quality of the instrument itself, but also on the quality of its strings. How to make high quality strings has been a concern among past generations of *qin* masters. Traditionally, strings are made of silk. In recent years, nylon wrapped metal strings (to be called "metal strings" in the rest of this paper) have been developed. Nowadays, most *qin* players in Mainland China use metal strings, whereas some *qin* players in Hong Kong and Taiwan insist on using silk strings. Both metal strings and silk strings have their own advantages and disadvantages, and there have been disputes over the choice.[2] As the production of sound from the string is a physical phenomenon, the properties of the string as well as the characteristics of its sound production can be investigated from a physical perspective. The first author of this paper, Tse Chun-yan, has written two articles on the physics of *qin* strings more than 10 years ago, one on longitudinal vibration of the strings, and the other on relative tensions of silk strings.[3] This article will focus on longitudinal vibration, using computer sound analysis to review and update the previous discussions.

Problem of the Metallic Noise with Metal Strings

Compared with silk strings, the sound of metal strings is brighter, and less friction sound is produced when the left finger glides on metal strings. However, the major problem of metal strings is the presence of a high pitched and loud metallic noise on some strings and some instruments. This metallic noise occurs mainly in open string notes, especially when the notes are played loudly. This may sometimes occur in stopped notes as well. This metallic noise sounds piercing, like striking metal, and disturbs the harmonious tone and tranquil mood of *qin* music. In 1999, Tse Chun-yan in his paper on longitudinal vibrations of *qin* strings, commented that this metallic noise should be related to longitudinal vibration of the strings.[4] With careful analysis of auditory findings, the following conclusions were reached in that paper:

1. The pitch of the metallic noise is little affected by adjusting the tension and fundamental pitch of the string. There may be variation of plus or minus a minor 2^{nd} only.

2. The pitch of the metallic noise is very close to the pitch of the sound produced by lightly rubbing the string longitudinally with fingers. The latter pitch is not affected by variations in the tension of the string.

3. When the commonly used metal strings produced in Beijing are mounted on a *qin* with effective length of 118cm, the pitches of the sound on rubbing longitudinally the 1^{st} and 2^{nd} strings are similar, and slightly higher than D6(d'''). The pitch of the metallic noise on these 2 strings, regardless of the tension of the string, is very close to this. The pitches of the sound on rubbing longitudinally the 3^{rd} to 7^{th} strings are also similar, and slightly higher than $F^{\#}6$($f^{\#}$'''). The pitch of the metallic noise on these 5 strings, regardless of the tension of the string, is again very close to this.

4. Although variation of the tension of the string does not affect the pitch of the sound on longitudinal rubbing, it affects the amplitude of the metallic noise. Mounted on a *qin* with effective length of 118cm, if the 2^{nd} string is tuned to D2, the metallic noise of the open string sound is loudest. On tuning the string gradually higher, when the 4^{th} octave of the fundamental pitch becomes higher than the pitch of the longitudinal rub, the amplitude of the metallic noise will decrease. On tuning the string gradually lower, the amplitude of the metallic noise will also decrease, but would increase again when the fundamental pitch comes close to C2.

5. The plucking position on the string will also affect the amplitude of the metallic noise. For an effective length of 118cm and with the 2^{nd} string tuned to the standard pitch D2, if the string is plucked at 1/8 distance from the bridge (the position of the 1^{st} stud), the metallic noise becomes softer. With the 1^{st} string tuned to the standard pitch C2, if the string is plucked at 1/9 distance from the bridge (to the right of the 1^{st} stud), the metallic noise becomes softer. The

harmonic notes at these two positions are both D5(d"),[5] which is close to an octave below the pitch of the longitudinal rub.

Tse Chun-yan pointed out in his paper that, according to reference sources, the sound on longitudinally rubbing the string, mentioned in point 2 above, arises from its longitudinal vibration. According to reference sources,[6] the frequency of this longitudinal vibration is not related to the tension of the string, but is inversely proportional to the length of the string, inversely proportional to the square root of the density of the string, and proportional to the square root of the elasticity of the string. For the same brand of strings, the similarity of pitches of the longitudinal rub at the 1st and 2nd strings indicates that the structures of the 1st and 2nd strings are similar, thus a similar density and elasticity. The similarity of pitches of the longitudinal rub at the 3rd to 7th strings indicates that the structures of the 3rd to 7th strings are similar, thus a similar density and elasticity. Tse postulated in his paper that, as the pitch of disturbing metallic noise is similar to that of the longitudinal rub, the former could be related to longitudinal vibration. Tse further postulated that, when a higher partial of the string sound comes near an octave below the pitch of longitudinal rub, the two pitches interact, resulting in resonance and an increase in amplitude of the metallic noise. When the plucking position is near a node of this partial, the amplitude of the partial would decrease,[7] reducing the effect on longitudinal vibration and thus reducing the amplitude of the metallic noise.

When his paper was published, Tse Chun-yan did not have physical measurements to support his conclusions, and there were no theories to support his postulations.

Review of Recent Academic Literature

In recent years, there began to have international journal articles discussing longitudinal vibration on other musical instruments, mainly on the piano. In 1999, H. A. Conklin pointed out that longitudinal vibrations of piano strings produce partials not found in transverse vibrations.[8] He called these "phantom partials". The frequency of a phantom partial is equal to twice the frequency of a transverse partial ($2f_n$), or the sum of two transverse partials ($f_m + f_n$), and the latter is mostly of two adjacent transverse partials ($f_n + f_{n+1}$). Because of stiffness of piano strings, there is "inharmonicity", which means that the frequencies of transverse partials of a piano string deviate from multiples of the fundamental frequency, being higher than the latter. Thus, ($2f_n$) and ($f_m + f_n$) would not be identical to transverse partials, and would be shown up among the transverse partials in spectrograms.

In their paper of 2005, B. Bank and L. Sujbert provided physical and mathematical explanations to the findings of Conklin.[9] Putting aside complex mathematical formulae, discussions in the paper that may be understood by lay readers include:

1. "Phantom partials" arise from forced response of longitudinal vibration. The forced response may occur concurrently with a free response.
2. If the frequency of the forced response of the longitudinal vibration is close to the frequency of its free response, resonance may occur, leading to a marked increase in its amplitude.
3. The free response decays rapidly. The pitch of longitudinal vibration which is readily heard arises from the forced response.
4. If the frequency of the forced response (phantom partial) is close to the frequency of the first mode of the free response, this phantom partial originates mainly from two adjacent transverse partials, f_n and f_{n+1}.
5. In addition to its value towards the study of piano acoustics, the research is of value to the study of acoustics of other string instruments, e.g. the guitar. The understanding of the origin of longitudinal vibration would assist in computer generation of sound of string instruments, and assist in improving the construction of pianos and guitars for better sound quality.

Other relevant papers include:

1. H. A. Conklin in 1996 pointed out the effect of longitudinal vibration on sound quality of the piano;[10]

2. H. Penttinen in 2006 argued that, in computer generation of the *qin* sound, one has to take into account its "phantom partials";[11]

3. Malashin in 2007 discussed forced longitudinal vibrations of prestretched flexible deformable strings; [12]

4. Bank in 2010 discussed the audibility of longitudinal vibrations of piano strings.[13]

Sound Analysis of Open String Tones of Metal Strings with Metallic Noise

In order to provide objective evidence for the relationship between the metallic noise of a *qin* string and its longitudinal vibration, the second author, Wong Chun-fung studied a *qin* with prominent metallic noise on metal strings.[14] The software used was Raven Pro 1.4 produced by Cornell Lab of Ornithology (http://www.birds.cornell.edu/brp/raven/RavenOverview.html). Sound was recorded by a digital recorder, Sony PCM-M10, with a condenser microphone, Takstar PCM-5550, placed at around 50cm beyond the 5th stud. All plucks were made with the right middle finger plucking inwards, with amplitude of *f*. All samples were taken at 44 kHz sampling rate and stored with 16 bit uncompressed linear pulse-code modulation (LPCM) format. Spectrograms were generated by applying Fast Fourier transformation (FFT) with 0.3s wide triangular window function to sound samples, timed 0.3s after the string was plucked.[15]

With the 2nd string tuned to D2 and the 1st string tuned to near C2,[16] the spectrograms of the open string notes are shown below.[17] The strings were plucked mid-way between the bridge and the 1st stud.

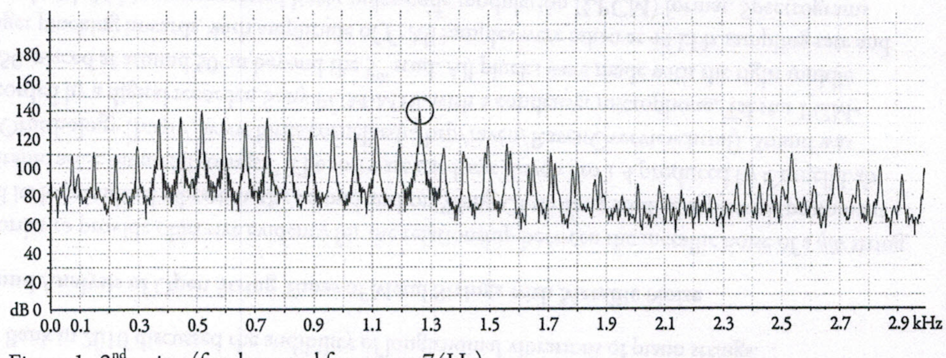


Figure 1 2nd string (fundamental frequency 74Hz)

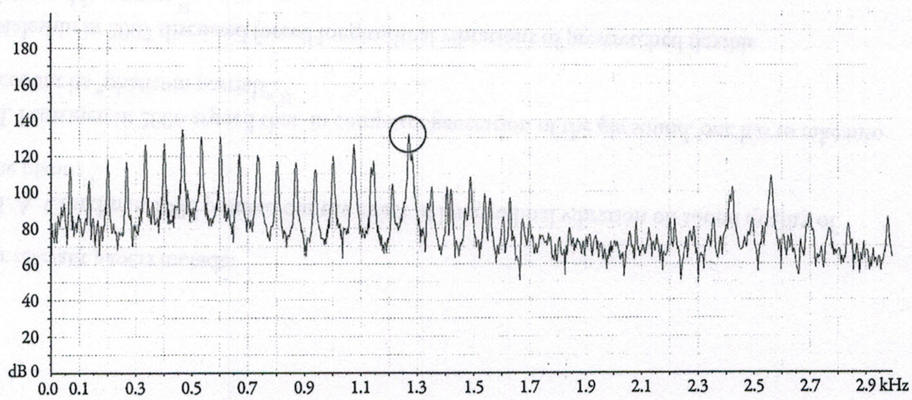


Figure 2 1st string (fundamental frequency 66.7Hz)

From the two figures, one can see that f_{17} of the 2nd string note (red circle at Figure 1) and f_{19} of the 1st string note (red circle at Figure 2) are very loud, louder than all the partials other than f_7. Their frequencies are 1262Hz and 1269Hz respectively, both being D$^{\#}$6, a tone not consonant with the fundamental tone. This should be the metallic noise that we hear.

The frequencies of f_1 to f_{10} according to Figures 1 and 2 are tabulated together with the theoretical frequencies of harmonic notes. The results are as follows.

2nd string as D2:

	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}
Frequency of harmonics (Hz)	74	148	222	296	370	444	518	592	666	740
Frequency according to Figure 1 (Hz)	74	148	222	296	369	445	519	593	667	743

1st string close to C2:

	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}
Frequency of harmonics (Hz)	66.7	133.4	200.1	266.8	333.5	400.2	466.9	533.6	600.3	667
Frequency according to Figure 2 (Hz)	66.7	133	200	266	333	401	467	534	600	669

From the above tables, one can see that the lower partials are very close to the theoretical frequencies of harmonics. This means the inharmonicity of the *qin* string is very small. In Figures 1 and 2, bifurcation of the peaks, representing separation of the phantom partials and the transverse partials, is evident only from f_{20} onwards. This means, the difference between the frequency of the longitudinal vibration ($2f_n$) or ($f_m + f_n$) and the frequency of transverse vibration is evident only among high partials. Therefore, at the position of the loud metallic noise at f_{17} and f_{19}, the transverse and longitudinal vibration frequencies overlap. The question whether the metallic noise of these two strings is related to longitudinal vibration or not has to be further investigated along the following lines:

1. The relationship between the metallic noise and the pitch of the free longitudinal vibration;
2. The relationship between the metallic noise and the fundamental tone of the string;
3. The relationship between the metallic noise and the plucking position by the right finger.

The Pitch of the Free Longitudinal Vibration

The pitch produced by lightly rubbing the string longitudinally arises from the free longitudinal vibration of the string. There is academic discussion on the physics of sound production on longitudinal rubbing of metallic rods.[18] For the *qin* used in the above-described sound testing, the rubbing sound of the 1st and 2nd strings is very close to D$^{\#}$6.[19] However, because the sound is very soft, it cannot be analysed with the software. Yet, when one turns the tuning peg slightly, the string produces a short clicking sound having the same pitch as that of the longitudinal rub. This should also be produced by longitudinal vibration. Wong Chun-fung analysed this short clicking sound of the 1st and 2nd strings, and found that both are 1271Hz, very close to the metallic noise shown in Figures 1 and 2. This supports the postulation that the metallic noise arises mainly from longitudinal vibration.

The Relationship between the Metallic Noise and the Fundamental Tone of the String

According to the paper of Bank in 2005, when there is a forced longitudinal vibration with a frequency close to the frequency of the first mode of the free longitudinal vibration, the former originates mainly from two adjacent transverse partials, f_n and f_{n+1}. According to the testing described above, on the 2nd string with the fundamental tone at D2, the frequency of the metallic noise is 1262Hz, which is D$^{\#}$6, and is close to the free longitudinal vibration frequency 1271Hz. Therefore, if the metallic noise of the 2nd string arises from longitudinal vibration, it should mainly originate from two adjacent transverse partials f_n and f_{n+1}. According to table 1, these two adjacent partials are f_8 and f_9. Their sum $(f_8 + f_9)$ is (593 + 667) Hz = 1260Hz, close to the frequencies of the free longitudinal vibration and the metallic noise.

Wong Chun-fung then repeated the testing with the fundamental tone of the 2nd string gradually tuned downwards. The results showed that, when $(f_8 + f_9)$ gradually deviates from the free longitudinal vibration frequency, and when the pitch of the metallic noise f_{17} is lowered correspondingly, the amplitude of metallic noise becomes softer. On the other hand, f_{15} becomes louder. Because f_{15} is a fifth above the loud f_{10} (which is a major 3rd and two octaves above the fundamental), the pitch is more consonant with the overall sound of the vibrating string. Thus, the sound is subjectively less noisy. Please see Figures 3 and 4 for details:

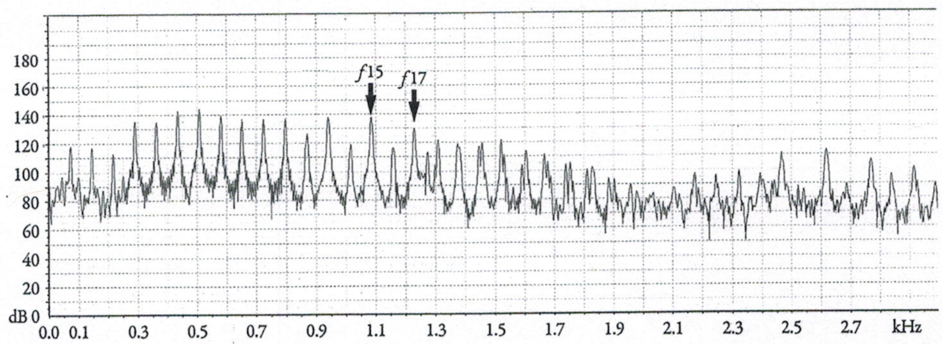


Figure 3 (fundamental frequency 72Hz)

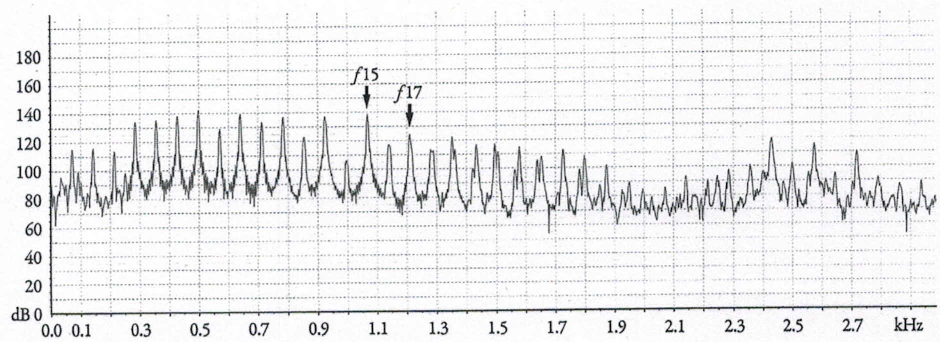


Figure 4 (fundamental frequency 70Hz)

However, when the fundamental tone is close to C2, the metallic noise becomes loud again. Please see Figures 5 and 6 for details:

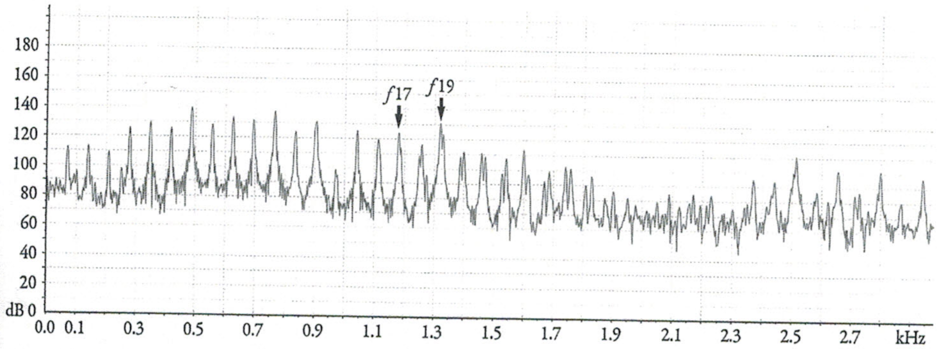


Figure 5 (fundamental frequency 68Hz)

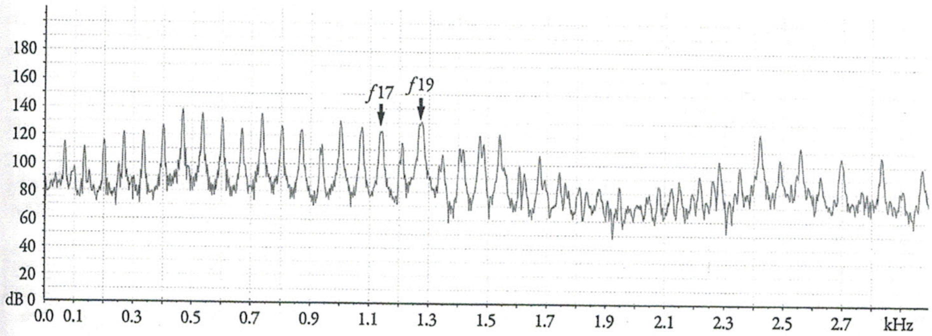


Figure 6 (fundamental frequency 66.7Hz)

But now, the loud metallic noise is f_{19} and not f_{17}. At first, it is higher than D$^{\#}$6. In Figure 5, its frequency is 1319Hz, which is E6. When the fundamental tone is close to C2, in Figure 6, it is 1271Hz, close to D$^{\#}$6. This is equal to the free longitudinal vibration frequency, and very close to the frequency of the metallic noise of the 2nd string tuned to D2.

The frequencies (Hz) of the partials at Figure 6 are as follows:

f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}
66.7	133	200	266	333	399	467	534	600	669

Here, $(f_8 + f_9)$ has decreased to 1134Hz, but $(f_9 + f_{10})$ is 1269Hz, very close to the frequencies of the free longitudinal vibration and the metallic noise at f_{19}.

Wong Chun-fung then gradually tuned the string upwards above D2. When $(f_8 + f_9)$ gradually deviates from the free longitudinal vibration frequency,[20] and when the pitch of the metallic noise f_{17} is raised correspondingly, the amplitude of metallic noise becomes softer. But f_{15}, with its frequency very close to $(f_7 + f_8)$, becomes louder. When the fundamental tone is above E2, the loud f_{15} in Figure 9 is 1261Hz, close to the free longitudinal vibration frequency. Because f_{15} is a fifth above the loud f_{10} (which is a major 3rd and two octaves above the fundamental), the pitch is more consonant with the overall sound of the vibrating string. Thus, the sound is subjectively less noisy. Please see Figures 7 to 9 for details:

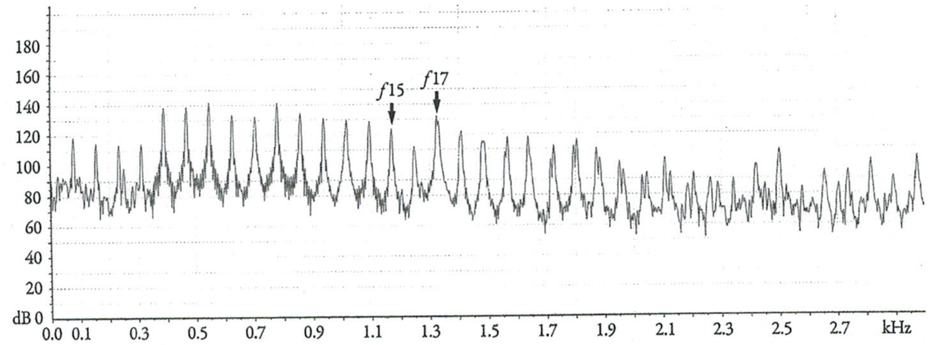


Figure 7 (fundamental frequency 78Hz)

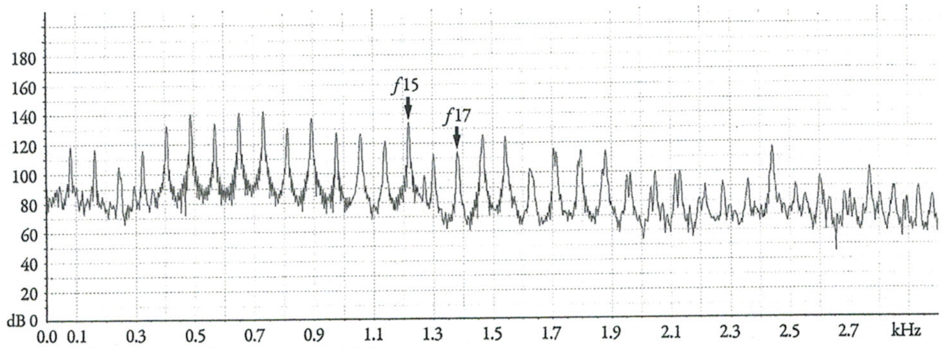


Figure 8 (fundamental frequency 81Hz)

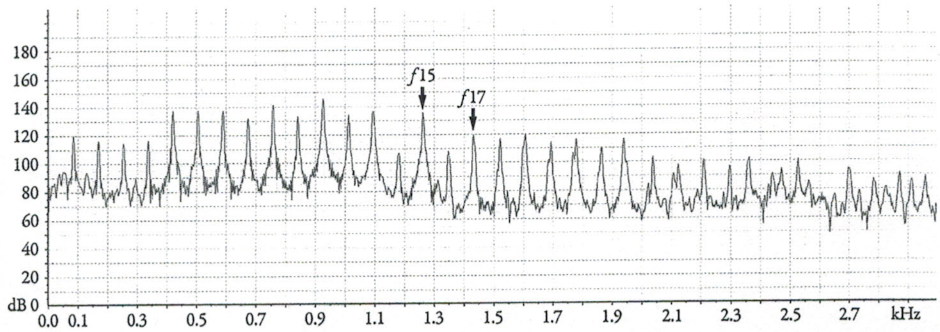
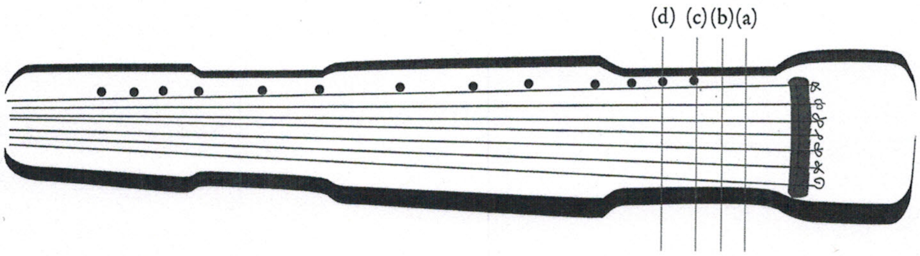


Figure 9 (fundamental frequency 84Hz)

One can see from the above that, if there is a sum $(f_n + f_{n+1})$ close to the free longitudinal vibration,[21] and the frequency is not consonant with the fundamental tone, the metallic noise is loudest and the frequency of the metallic noise is very close to $(f_n + f_{n+1})$. This supports the postulation that the metallic noise arises mainly from longitudinal vibration. If the 2nd string is tuned to D2, the metallic noise of the open string tone arises mainly from the longitudinal vibration $(f_8 + f_9)$ generated by the transverse vibration partials f_8 and f_9, with the pitch D$^\#$6. If the 2nd string is tuned close to C2, the metallic noise of the open string tone arises mainly from the longitudinal vibration $(f_9 + f_{10})$ generated by the transverse vibration partials f_9 and f_{10}, again with the pitch D$^\#$6. Similarly, if the 1st string is tuned to C2, the metallic noise of the open string tone arises mainly from the longitudinal vibration $(f_9 + f_{10})$ generated by the transverse vibration partials f_9 and f_{10}, again with the pitch D$^\#$6.

The Relationship between the Metallic Noise and the Plucking Position by the Right Finger

When the right finger plucks at the node of a transverse vibration partial, the vibration of this partial would be decreased. Thus, with the 2nd string tuned to D2, if the metallic noise of the open string tone arises mainly from the longitudinal vibration $(f_8 + f_9)$ generated by the transverse vibration partials f_8 and f_9, the metallic noise would decrease in amplitude when the pluck is at the node of f_8 or f_9.[22] This is similar for the 1st string. With the 1st string tuned to C2, if the metallic noise of the open string tone arises mainly from the longitudinal vibration ($f_9 + f_{10}$) generated by the transverse vibration partials f_9 and f_{10}, the metallic noise would decrease in amplitude when the pluck is at the node of f_9 or f_{10}.[23] Corresponding tests were carried out and the results are as follows:



Plucking position (a) is mid-way between the bridge and the 1st stud
Plucking position (b) is at the 1st "un-marked" stud to the right of the 1st stud
Plucking position (c) is at the 1st stud
Plucking position (d) is at the 2nd stud

The following table shows the results of testing when the 1st string is tuned close to C2 (66.7Hz). To facilitate reading, only f_{10} to f_{20} are shown (frequency in Hz): [24]

Plucking position		f_{10}	f_{11}	f_{12}	f_{13}	f_{14}	f_{15}	f_{16}	f_{17}	f_{18}	f_{19}	f_{20}
a	Frequency	669	733	804	868	937	1001	1075	1136	1210	1269	1350
	dB	120.9	120.4	116.4	115.8	111.7	119.2	126	113	103.9	131.5	102.1
b	Frequency	669	735	804	872	940	1001	1077	1145	1212	1271	1350
	dB	121.7	132	123.7	125.2	125.3	121.3	131.1	125.7	111.5	122	103.2
c	Frequency	671	737	805	874	940	1007	1077	1145	1212	1273	1352
	dB	125.6	133.6	124.6	120.2	123.5	108.2	110	117.1	107.4	129.5	107.4
d	Frequency	671	737	805	872	940	1009	1077	1136	1204	1273	1352
	dB	127.4	125.8	108	123.5	128.6	123.7	129.4	129.8	112.7	132.4	103.4

The following table shows the results of testing when the 2nd string is tuned to D2 (74Hz). To facilitate reading, only f_{10} to f_{20} are shown (frequency in Hz):

Plucking position		f_{10}	f_{11}	f_{12}	f_{13}	f_{14}	f_{15}	f_{16}	f_{17}	f_{18}	f_{19}	f_{20}
a	Frequency	743	818	892	966	1042	1114	1193	1262	1343	1411	1485
	dB	133.9	126.1	124.2	125.7	123.3	126.6	115.9	138.8	112.7	110.1	109.9
b	Frequency	743	817	894	968	1044	1118	1193	1262	1345	1411	1485
	dB	127	122.6	128.9	133.8	136.5	134.3	121.1	117.7	107.9	97.6	92.2
c	Frequency	744	822	896	972	1046	1121	1197	1265	1349	1415	1489
	dB	133	125.7	132.1	132.4	134.5	126	102.5	120.7	108.5	111.6	105.7
d	Frequency	743	818	894	968	1044	1118	1193	1262	1345	1413	1487
	dB	136.3	126	117.3	125.7	135.4	133.4	124.9	129.8	100.2	112.5	110.6

The testing results show that, f_{19} of the open string note of the 1st string is loudest when the string is plucked mid-way between the bridge and the 1st stud (position a). It is softest when plucked to the right of the 1st stud (position b), and becomes loud again when plucked at the 1st stud (position c) and at the 2nd stud (position d). Similarly, f_{17} of the open string note of the 2nd string is loudest when the string is plucked mid-way between the bridge and the 1st stud (position a). But it is softest when plucked to the right of the 1st stud (position b) and at the 1st stud (position c), and becomes loud again when plucked at the 2nd stud (position d). These results support the postulation that the metallic noise mainly arises from longitudinal vibration.

Metallic Noise in Stopped Notes and in the 3rd to 7th Open String Notes of Metal Strings

The testing described above concentrated on the 1st and 2nd open string notes. The situation in stopped notes and in the 3rd to 7th open string notes should be similar. Because the free longitudinal vibration frequency is inversely proportional to the length of the string, the pitch of the metallic noise of a stopped note would change with the stopped position. The relationship between the right finger plucking position and the metallic noise would also change with the stopped position. Regarding the 3rd to 7th open string notes, the metallic noise depends on the relationship between the free longitudinal vibration frequency and the fundamental tones. In order to appreciate the details, further studies are needed.

Longitudinal Vibration of Silk Strings

Longitudinal vibrations are not limited to metal strings, but may occur also on silk strings. The free longitudinal vibration pitch of a silk string sounds much lower than that of a metal string. Thus, the forced longitudinal vibration resonating with the free longitudinal vibration should have a lower frequency and more consonant with the overall sound. Therefore, the metallic noise of silk strings are often less prominent. In order to appreciate the details, further studies are needed.

Discussion

The results of computer analysis of the metallic noise are compatible with the auditory findings reported in the 1999 paper of Tse Chun-yan, and provide objective evidence for his conclusions. In 1999, Tse postulated that the metallic noise arises from longitudinal vibration. This postulation is supported by the current study. In 1999, Tse further postulated that "when a higher partial of the string sound comes near an octave below the pitch of longitudinal rub, the two pitches interact, resulting in resonance and an increase in amplitude of the metallic noise." This turns out to be not exactly true. The correct situation should be, when the sum of frequencies of two adjacent transverse vibration partials ($f_n + f_{n+1}$) is close to the free longitudinal vibration frequency, resonance occurs leading to a loud metallic noise from forced longitudinal vibration.

As to how to deal with this metallic noise, the suggestions of Tse Chun-yan in 1999 were basically appropriate. With the current findings, the following are updated suggestions:

1. If the metallic noise is too loud, plucking at the node of the harmonic note close to the pitch an octave lower than the free longitudinal vibration pitch may reduce the loudness of the metallic noise. Usually, the metallic noise of the 1st and 2nd open string notes of metal strings is near D#6. For the 2nd string, it is better to pluck at the 1st stud or the 1st un-marked stud (to the right of the 1st stud), which are the nodes of f_8 and f_9 respectively. For the 1st string, it is better to pluck at the 1st un-marked stud or the 2nd un-marked stud (further to the right of the 1st stud), which are the nodes of f_9 and f_{10} respectively.

2. If the metallic noises of one or two strings are too loud, and when there is no necessity to tune the strings to standard pitches, one can tune the whole set of strings higher or lower by a quarter tone to a semitone. The metallic noise of the respective strings may then be reduced. However, the metallic noise of some other strings may increase as a result. Therefore, one should assess the overall situation before deciding how to tune the strings.

3. For the same instrument, if the strings are tuned slightly higher or lower, the loudness of the metallic noise may change. This may affect one's judgment of the quality of the instrument. Therefore, when assessing the quality of a *qin*, one should pay attention to the effect of its tuning to its sound quality.

4. For the same brand of strings, the loudness of the metallic noise is different on different instruments. This indicates that the instrument itself has significant influence on longitudinal

vibration. This could be a difference in resonance, or a difference in response of the bridge to longitudinal vibration. It is hoped that *qin* makers can find ways to reduce the metallic noise in the instrument. At the same time, it is also hoped that string makers can make fine adjustments in the material or the making of the strings, so that the free longitudinal vibration pitch pairs up better with standard tuning, reducing the dissonant effect.[25]

A question at another level is whether the metal string or the silk string is better. The silk string is traditional, whereas the metal string is a modern development. Both the currently available silk and metal strings have their own advantages and disadvantages. Different *qin* players have their own preference and insistence, involving complex value and aesthetic factors. Detailed discussion of this is beyond the scope of this paper. The authors just consider that the qualities of both the currently available silk and metal strings have rooms for improvement. Also, both silk and metal strings should have their own important position in the contemporary world of *qin* music.

1 Translation of the first two lines of the poem *Feiqin* (*The Deserted Qin*), by the Tang Dynasty poet Bai Juyi.

2 For a discussion of the making of *qin* strings in recent years, one may read the article by Cheng Gongliang, "*Qin* strings in the past fifty years," in *Yinyue Aihaozhe*, no. 6 (2009), pp. 44–47.

3 Tse Chun-yan, "Sound quality of the *qin* and longitudinal vibration of its strings," in *Qixianqin Yinyue Yishu*, no. 5 (1999), pp. 29–30; Tse Chun-yan, "The relative tension of silk strings of the *qin*," in *Qixianqin Yinyue Yishu*, no. 8 (2001), pp. 37–39.

4 Sound emitted from a vibrating string mainly comes from vibrations perpendicular to the axis of the string, called transverse vibrations. A longitudinal vibration is a periodic change in the displacement of elements of the string along its axis. Longitudinal vibrations are not main components of a string sound, but would affect the overall tone quality of the sound.

5 The harmonic note at 1/8 position (the 1st stud) of the 2nd string is higher than the fundamental pitch by 3 octaves. The harmonic note at 1/9 position (the first "un-marked" stud to the right of the 1st stud) of the 1st string is higher than the fundamental pitch by 3 octaves plus one major 2nd. Thus, both are D5(d'').

6 Jess J. Josephs, *The Physics of Musical Sound* (Princeton: D. Van Nostrand Company, 1967), pp. 91–92. Another source is at Neville H. Fletcher and Thomas D. Rossing, *The Physics of Musical Instruments* (New York: Springer-Verlag, 1991), pp. 53–54.

7 Nodes of a transverse vibration are relatively motionless. If the finger plucks at the position of a node, the corresponding partial would be suppressed.

8 H. A. Conklin, "Generation of partials due to nonlinear mixing in a stringed instrument," in *J. Acoust. Soc. Am.* 105 (1999), pp. 536–545.

9 B. Bank and L. Sujbert, "Generation of longitudinal vibrations in piano strings: From physics to sound synthesis," in *J. Acoust. Soc. Am.* 117 (2005), pp. 2268–2278. An earlier paper by the same authors on the topic is B. Bank and L. Sujbert, "Modeling the longitudinal vibration of piano strings," in *Proc. Stockholm Music Acoust. Conf.*, Stockholm, Sweden (2003), pp. 143–146.

10 H. A. Conklin, "Design and tone in the mechanoacoustic piano, part III: Piano strings and scale design," in *J. Acoust. Soc. Am.* 100 (1996), pp. 1286–1298.

11 H. Penttinen, J. Pakarinen, V. Välimäki, M. Laurson, H. Li, and M. Leman, "Model-based sound synthesis of the *guqin*," in *J. Acoust. Soc. Am.* 120 (2006), pp. 4052-4063.

12 A.A. Malashin, "Forced longitudinal vibrations of prestretched flexible deformable strings at the frequencies of transverse vibrations," in *Doklady Physics* 52, no. 9 (2007), pp. 496–498.

13 B. Bank and H.-M. Lehtonen, "Perception of longitudinal components in piano string vibrations," in *J. Acoust. Soc. Am.* 128, no. 3 (2010): EL117-123.

14 The effective length of the instrument is 110.2cm, shorter than the one studied by Tse Chun-yan in 1999 by 7.8cm.

15 Sound samples were taken 0.3s after the string was plucked, in order to avoid the initial noise on plucking the string.

16 Because the purpose of the study is to investigate the metallic noise, the fundamental pitch of the 1st string chosen is one close to C2 and with the loudest metallic noise. This is slightly higher than C2 by 1.3Hz.

17 The dB shown in the figures are relative ones and do not represent the actual amplitude. This applies to dB shown in other figures and tables in this paper.

18 Steven Errede, "The Physics of a Longitudinally Vibrating Metal Rod," in *UIUC Physics 193POM/498POM Physics of Music/Musical Instruments*, Department of Physics, University of Illinois, http://ebookbrowse.com/longitudinally-vibrating-singing-rod-pdf-d441976184 (accessed April 2013).

19 The pitch of the longitudinal rub described in Tse Chun-yan's 1999 paper was slightly higher than D6. This time, as the *qin* is slightly shorter (please see footnote 14), the pitch of the rub is higher.

20 As discussed above, when the fundamental tone of 2nd string is D2, $(f_8 + f_9)$ is close to its free longitudinal vibration frequency.

21 Please note that the loud f_{15} at Figures 3 and 4 are not explained by proximity to the free longitudinal vibration frequency. The authors are not able to provide a theoretical explanation to this. However, this does not contribute too much to the metallic noise that we hear.

22 The node of f_8 furthest towards the right is at 1/8 of the string from the bridge (i.e. the 1st stud). The node of f_9 furthest towards the right is at 1/9 of the string from the bridge (i.e. the 1st "un-marked" stud to the right of the 1st stud).

23 The node of f_{10} furthest towards the right is at 1/10 of the string from the bridge (i.e. the 2nd "un-marked" stud to the right of the 1st stud).

24 dB in the table indicate relative amplitudes and not absolute ones. Please also note that there are slight bifurcations of the peaks of the higher partials. The figures in the table show the lower frequency spikes, because when the longitudinal vibration partial ("phantom partial") separates from the transverse vibration partial, the former is the lower frequency one.

25 Piano makers have already paid attention to this point in 1990s. Please see H. A. Conklin, "Design and tone in the mechanoacoustic piano, part III: Piano strings and scale design," in *J. Acoust. Soc. Am.* 100 (1996), pp. 1286–1298.